

Discrete Math

9-12 Grade

Science Based Math

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Executive Summary

Unit One: Electrical Resistance

Days Required: 5

Lesson Objective: Explain, represent, and calculate the resistance of a circuit.

Unit Two: Genetics & Probability

Days Required: 5

Lesson Objective: Understand the idea of likelihood. Manipulate the mathematical formulas for probability and relative frequency. Calculate possible outcomes. Make predictions of probability of an event.

Unit Three: Blood Typing with Venn Diagrams

Days Required: 2

Lesson Objective: Students will understand how the 2 antigens of blood work together with the RH factor to form the different blood types.

Unit Four: Password Security

Days Required: 3

Lesson Objective: To learn what permutations are and how to calculate permutations.

Sample MCA Question

A group of health care providers consists of 4 doctors, 3 dentists, and 5 nurses. How many combinations of 2 health care providers of different types are possible?

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Electrical Resistance (Discrete)

Lesson Objective: Explain, represent, and calculate the resistance of a circuit.

Days of Instruction: 5

Day 1 - Pretest. Launch/Explore

Day 2 - Euler circuits/schematics intro.

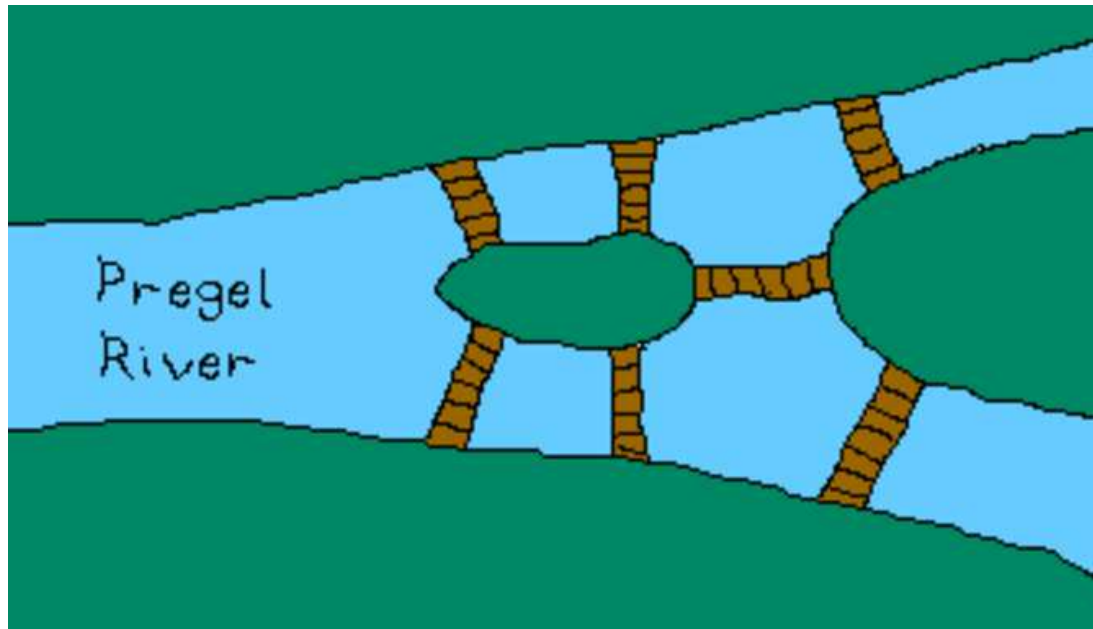
Day 3 - Hands on lab. Measuring resistance of parallel & series circuits with ohmmeter.

Day 4-5 - Abstract method of calculating resistance. Sample problems below. Posttest.

Minnesota State Standard(s):

Math	Algebra	Understand the concept of function, and identify important features of functions and other relations using symbolic and graphical methods where appropriate.	9.2.1.7	Understand the concept of an asymptote and identify asymptotes for exponential functions and reciprocals of linear functions, using symbolic and graphical methods.
Math	Algebra	Generate equivalent algebraic expressions involving polynomials and radicals; use algebraic properties to evaluate expressions.	9.2.3.4	Add, subtract, multiply, divide and simplify algebraic fractions.
Physical Science	Energy	Electrons respond to electric fields and voltages by moving through electrical circuits and this motion generates magnetic fields.	9P.2.3.2.2	Explain and calculate the relationship of current, voltage, resistance and power in series and parallel circuits. For example: Determine the voltage between two points in a series circuit with two resistors.

Launch: The Seven Bridges of Konigsberg. My doctors said I need to exercise more so exercise of choice is walking. I want to walk across all the bridges of Konigsberg only once and end up where I started (my house).



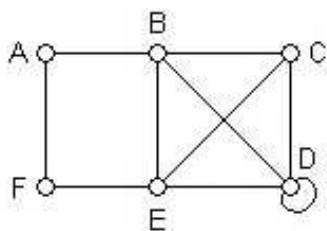
<http://www.oocities.org/zigadabooga/Pictures/konigsberg.gif>

Explore: Show the students the map (see below) of Konigsberg and let them explore. Then led them to the idea of a vertex edge graph by asking them what really is important (each land mass - not the shape of the land mass-, and the bridges that connect the land masses - not the location of the bridges-)

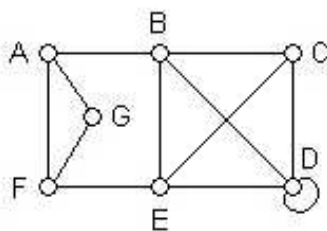
Put up some examples and ask the students if they can travel over all the edges and get back to where you started (Euler Circuit), travel all the edges but not get back to where you started (Euler Path), not travel all the edges. Asked them what graph characteristics of the graph form an Euler Circuit or an Euler Path?

http://web.mit.edu/urban_or_book/www/book/chapter6/images6/fig6.14.gif

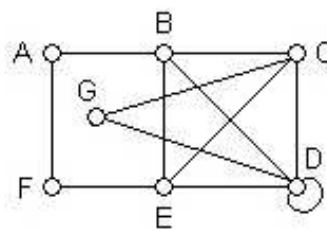
Which of the following graphs has an Euler circuit?



Graph 1



Graph 2



Graph 3

- A. Graph 3 only
 B. Graph 2 only
 C. Graph 1 only
 D. Graphs 1 and 3
 E. none of the above

<https://www.math.ku.edu/~jmartin/courses/math105-F14/h.png>

Share: Students will share verbally in class and go to the whiteboard and share their findings and explain their work to the class.

Summarize: The teacher will summarize the main points of the lesson that Euler Circuits (all vertices have an even degree) and Euler Path (all vertices are even except two vertices are odd and the two odd vertices are your starting and ending locations).

Explore: Give students the following problems and ask them how they could solve them and share their work and thoughts at the board.

$$\frac{1}{4} + \frac{1}{3} = \frac{1}{r}$$

$$\frac{1}{5} + \frac{1}{x} = \frac{1}{2}$$

$$\frac{1}{r} + \frac{1}{3} = \frac{5}{2}$$

$$\frac{2x+4}{5x} = \frac{2}{x}$$

$$\frac{x}{3} = \frac{-2}{x+7}$$

$$2 = \frac{x+2}{x-3}$$

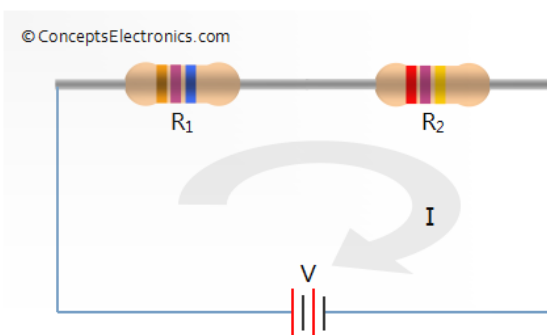
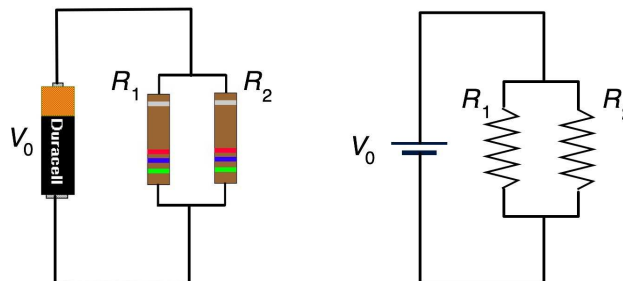
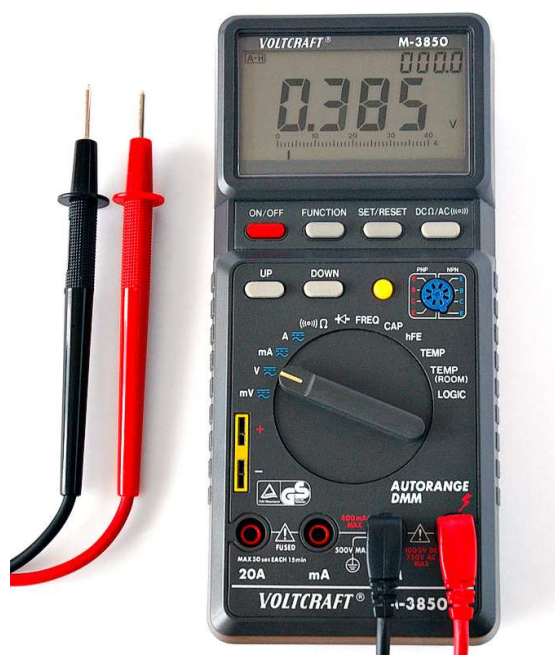
Share: Students will share verbally in class and go to the whiteboard and share their findings and explain their work to the class.

Summarize: The teacher will summarize the main points of the lesson of finding the least common multiple of the denominator, writing each fraction over that common denominator (the LCM of the denominators), then continue to solve the resulting equation.

Hands On: Measuring Resistance within Circuits. Series vs. Parallel

Series Circuit: Pick two resistors from your parts box in the 10,000W range. Measure the resistances using the Ohmmeter and make sure the values agree with the color code. Connect the resistors in series and measure the total resistance and see that this resistance agrees with what you expect from numerical calculation. Calculate the current using Ohm's law. Measure the value of the current in each resistor and make sure this agrees with the theoretical numerical value of the current. Calculate the voltage across each resistor using Ohm's law. Verify that these voltages are correct using the voltmeter attached to the circuit in the proper manner described above.

Parallel Circuit: Again, pick two resistors from your parts box in the 10,000W range. Measure the resistances using the Ohmmeter and make sure the values agree with the color code. Connect the resistors in parallel and measure the total resistance and see that this resistance agrees with what you expect from numerical calculation. Calculate the total current supplied by the battery using Ohm's law. Measure the value of the total current and make sure this agrees with the theoretical numerical value of the current. Calculate the current through each resistor using Ohm's law. Verify that these currents are correct using the ammeter attached to the circuit in the proper manner described above. Also, make sure the total current supplied by the battery is the sum of the currents through the resistors



Resistors R_1 and R_2 in series

Abstract Calculation of Resistance within a Circuit

The total resistance of a parallel circuit is NOT equal to the sum of the resistors (like in a series circuit).

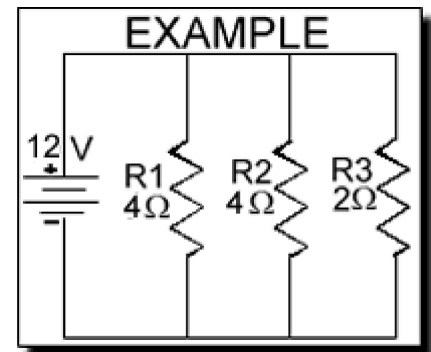
$$\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \dots = \frac{1}{R_{total}}$$

For as many resistors as you have.

For the problem to the right it would be

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{2} = \frac{1}{R_t}$$

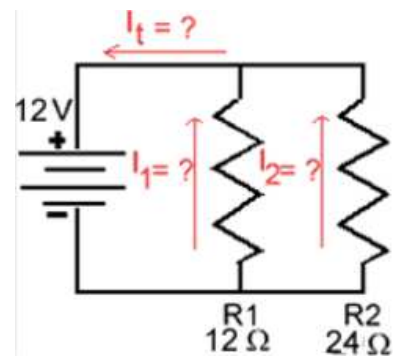
We get the total resistance to be 1 ohm.



For the problem at the right we see a 12 and 24 ohm resistor in parallel with a 12 volt source. First we figure out the total resistance of the circuit:

$$1/R_t = 1/12 + 1/24$$

$$R_t = 8 \text{ Ohms}$$

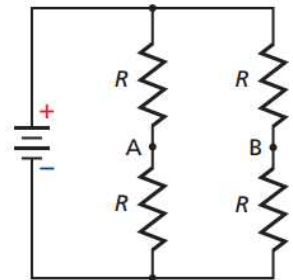


Sample Problems - Choose from the following to give to student groups. Have groups show their work on the board and discuss with the class.

The following problems come from <http://www.cttech.org/GOODWIN/academics/science-textbook/chap23.pdf>

1. Three $20\text{-}\Omega$ resistors are connected in series across a 120-V generator. What is the equivalent resistance of the circuit? What is the current in the circuit?
2. A $10\text{-}\Omega$, $15\text{-}\Omega$, and $5\text{-}\Omega$ resistor are connected in a series circuit with a 90-V battery. What is the equivalent resistance of the circuit? What is the current in the circuit?
3. A 9-V battery is in a circuit with three resistors connected in series.
 - a. If the resistance of one of the resistors increases, how will the equivalent resistance change?
 - b. What will happen to the current?
 - c. Will there be any change in the battery voltage?
4. A string of holiday lights has ten bulbs with equal resistances connected in series. When the string of lights is connected to a 120-V outlet, the current through the bulbs is 0.06 A .
 - a. What is the equivalent resistance of the circuit?
 - b. What is the resistance of each bulb?
5. Calculate the voltage drops across the three resistors in problem 2, and verify that their sum equals the voltage of the battery.
6. The circuit shown in Example Problem 1 is producing these symptoms: the ammeter reads 0 A , V_A reads 0 V , and V_B reads 45 V . What has happened?
7. Suppose the circuit shown in Example Problem 1 has these values: $R_A = 255\ \Omega$, $R_B = 292\ \Omega$, and $V_A = 17.0\text{ V}$. No other information is available.
 - a. What is the current in the circuit?
 - b. What is the battery voltage?
 - c. What are the total power dissipation and the individual power dissipations?
 - d. Does the sum of the individual power dissipations in the circuit equal the total power dissipation in the circuit? Explain.
8. Holiday lights often are connected in series and use special lamps that short out when the voltage across a lamp increases to the line voltage. Explain why. Also explain why these light sets might blow their fuses after many bulbs have failed.
9. The circuit in Example Problem 1 has unequal resistors. Explain why the resistor with the lower resistance will operate at a lower temperature.
10. A series circuit is made up of a 12.0-V battery and three resistors. The voltage across one resistor is 1.21 V , and the voltage across another resistor is 3.33 V . What is the voltage across the third resistor?

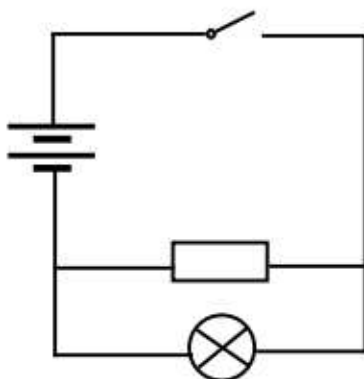
- 15.** Three $15.0\text{-}\Omega$ resistors are connected in parallel and placed across a 30.0-V battery.
- What is the equivalent resistance of the parallel circuit?
 - What is the current through the entire circuit?
 - What is the current through each branch of the circuit?
- 16.** A $120.0\text{-}\Omega$ resistor, a $60.0\text{-}\Omega$ resistor, and a $40.0\text{-}\Omega$ resistor are connected in parallel and placed across a 12.0-V battery.
- What is the equivalent resistance of the parallel circuit?
 - What is the current through the entire circuit?
 - What is the current through each branch of the circuit?
- 17.** Suppose that one of the $15.0\text{-}\Omega$ resistors in problem 15 is replaced by a $10.0\text{-}\Omega$ resistor.
- Does the equivalent resistance change? If so, how?
 - Does the amount of current through the entire circuit change? If so, in what way?
 - Does the amount of current through the other $15.0\text{-}\Omega$ resistors change? If so, how?
- 18.** A $150\text{-}\Omega$ branch in a circuit must be reduced to $93\ \Omega$. A resistor will be added to this branch of the circuit to make this change. What value of resistance should be used and how must the resistor be connected?
- 19.** A $12\text{-}\Omega$, 2-W resistor is connected in parallel with a $6.0\text{-}\Omega$, 4-W resistor. Which will become hotter if the voltage across them keeps increasing?
- 20. Circuit Types** Compare and contrast the voltages and the currents in series and parallel circuits.
- 21. Total Current** A parallel circuit has four branch currents: 120 mA , 250 mA , 380 mA , and 2.1 A . How much current is supplied by the source?
- 22. Total Current** A series circuit has four resistors. The current through one resistor is 810 mA . How much current is supplied by the source?
- 23. Circuits** A switch is connected in series with a 75-W bulb to a source of 120 V .
- What is the potential difference across the switch when it is closed (turned on)?
 - What is the potential difference across the switch if another 75-W bulb is added in series?
- 24. Critical Thinking** The circuit in **Figure 23-8** has four identical resistors. Suppose that a wire is added to connect points A and B. Answer the following questions, and explain your reasoning.
- What is the current through the wire?
 - What happens to the current through each resistor?
 - What happens to the current drawn from the battery?
 - What happens to the potential difference across each resistor?



■ **Figure 23-8**

The below problems come from https://ftp.yzu.edu.tw/nongnu/fhsst/Physics_Grade_10_Physics_Ch10_Electric_Circuits.pdf

- Using physical components, set up the physical circuit which is described by the circuit diagram below:



- Now draw a picture of the physical circuit you have built.
- Using physical components, set up a closed circuit which has one battery and a light bulb in series with a resistor.
 - Draw the physical circuit.
 - Draw the resulting circuit diagram.
 - How do you know that you have built a closed circuit? (What happens to the light bulb?)
 - If you add one more resistor to your circuit (also in series), what do you notice? (What happens to the light from the light bulb?)
 - Draw the new circuit diagram which includes the second resistor.
- Draw the circuit diagram for the following circuit: 2 batteries, a switch in series and 1 lightbulb which is in parallel with two resistors.
 - Now use physical components to set up the circuit.

Electrical Resistance Pre/Post Test

1. What is the resistance of two 50 ohm resistors wired in series?
2. What is the resistance of two 50 ohm resistors wired in parallel?
3. What is the resistance of one 25 ohm resistor and one 75 ohm resistor wired in parallel?
4. What is the resistance of one 25 ohm resistor and one 75 ohm resistor wired in series?

Genetics & Probability (Discrete)

Lesson Objective: After completing the lessons in the unit genetics and probability students will be able to:

1. Understand the idea of likelihood.
2. Manipulate the mathematical formulas for
3. probability and relative frequency.
4. Calculate possible outcomes.
5. Make predictions of probability of an event.

Days of Instruction: 5

Day 1 - Pretest. Launch/Explore.

Day 2 - Mr. Potato Head (Intro to permutations and combinations)

Day 3 - Permutation/combination questions in groups. Discuss student work as class.

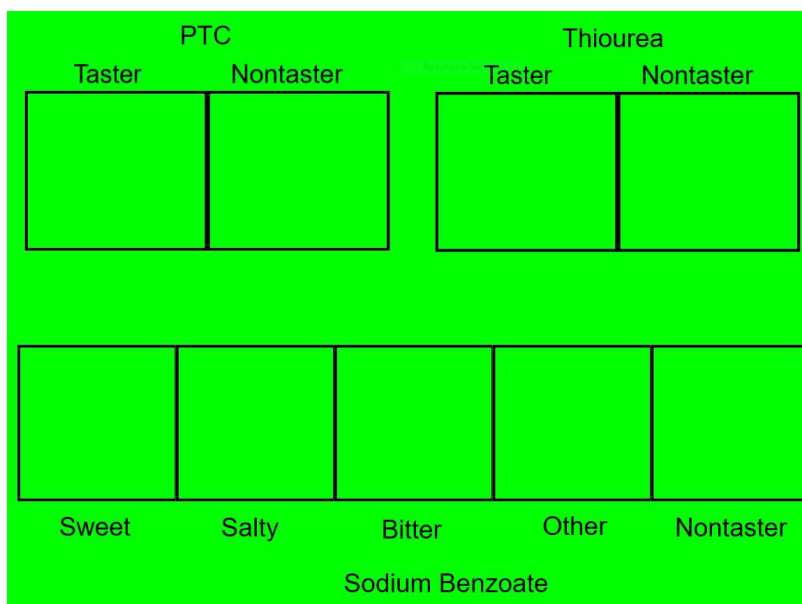
Day 4 - Intro to genetics and punnett squares. Real world combinations.

Day 5 - Real world combinations continued and Posttest

Minnesota State Standard(s):

Life Science	Evolution in Living Systems	Variation within a species is the natural result of new inheritable characteristics occurring from new combinations of existing genes or from mutations of genes in reproductive cells.	9.4.3.2.1	Use concepts from Mendel's laws of segregation and independent assortment to explain how sorting and recombination (crossing over) of genes during sexual reproduction (meiosis) increases the occurrence of variation in a species.
Math	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.1	Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.
Math	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.2	Calculate experimental probabilities by performing simulations or experiments involving a probability model and using relative frequencies of outcomes.
Math	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.5	Apply probability concepts such as intersections, unions and complements of events, and conditional probability and independence, to calculate probabilities and solve problems.
Math	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.7	Understand and use simple probability formulas involving intersections, unions and complements of events.

Launch: Begin this unit by discussing why people taste foods differently. For example, ask students to raise their hands if they like pickles. Then ask students to raise their hands if they do not like pickles. Document these numbers on the board. The students will most likely agree that they either love or hate pickles. Very few people will be neutral regarding this topic. You can ask this question about various foods. Ask students why this is? We are all the same species but yet have drastically different taste preferences. This will launch the discussion of the probability of a random person liking various foods and lays the groundwork for the following class experiment.



Explore: Before discussing genetics students need to have a firm understanding of probability regarding both permutations and combinations. To get students thinking about genetics and probability they will take part in a "Bio Tasters Lab". This lab involves each student tasting strips of paper with different chemicals. Students will then decide if they were able to taste the chemical or not. Because the ability to taste these various chemicals is an inherited trait, the ratios of tasters to nontasters can be used to launch a discussion about genetics and probability which the instructor can direct towards combinations and permutations (genotypes).

Share: Students will share verbally in class and go to the whiteboard and share their findings and explain their work to the class.

Summarize: The teacher will summarize the main points of the lesson focusing on the probability of students being tasters of various chemicals.

Fundamental Counting Principle

LAUNCH:

Consider having all the parts to a Mr. Potato Head set, in general, how can you find the number of possibilities for a Mr. Potato Head? Did anyone notice a relationship between your answer yesterday and the number of parts within your set?

EXPLORE:

Working in groups (3 or 4 students per group), students will complete exercises using their preferred strategy.

- You are ordering lunch. The only choice for an entrée is a hamburger. You need to choose chips or fries. The drink options are bottled water, lemonade, or apple juice. Find the number of possibilities for your lunch.
- You are planning activities for the weekend. On Saturday you can either go to the movies or to the mall. On Sunday you can choose to participate in one of the following sports events: basketball, soccer, flag football, or lacrosse. Find the number of possibilities for weekend activities.

SHARE:

Discuss as a class the strategies that groups used to find all the possibilities. Ask students to explain how they know that they have listed all possibilities and have students compare/critique the methods that are shared. Again ask, in general, how can you find the number of possibilities in situations like this. Ideally, there will be a few students that will be able to articulate that the number of possibilities is the same as the product of the choices for each option (or category). So in general, you can multiply the number of choices that each option can occur. If students struggle to articulate the principle, consider referring back to the Mr. Potato Head example to help them generalize the rule.

Present additional examples for group and class discussion.

- The store at your school wants to stock sweatshirts that come in four sizes (small, medium, large, x-large) and in two colors (red and white). How many different types of sweatshirts will the store have to stock?
- The call letters for all radio stations in the United States start with either a W (east of the Mississippi river) or a K (west of the Mississippi River) followed by three other letters that can be repeated. How many different call letters are possible?
- A restaurant offers a fixed-price dinner menu for \$30. The dinner consists of three courses, and the diner chooses one item for each course. There are 3 choices in the first course, 4 choices for the second course, and 2 choices for the third course.

SUMMARIZE:

The Fundamental Counting Principle allows you to multiply the number of choices that each option can occur.

Launch: I was doing some house cleaning yesterday and decided to clean out the game closet. I came across two dice. I wondered how many ways could I roll two dice and get double and what is the probability of rolling doubles?

Explore: Have students list all possible outcomes of rolling two dice (tree diagram, organized list, 6 by 6 table, ...) encourage students to try more than one method to verify that they have listed all the options. How do students know they have listed all the options? Is there a shorter way to figure out the total number of outcomes without list all the options? What is the probability of rolling doubles? A sum 7? A sum of 5? A sum of 2? A sum of 9? A sum of 3 or 7? Both dice even or doubles? Both dice even and double?

Share: Students will share their answer with work at the board while they verbally share their work to the class

Summarize: The teacher will summarize the main points of the lesson how to list all the outcome through an organized list, tree diagram, or table then follow that with explaining how to find the total number of outcomes without having to list out all the options (Fundamental Counting Principle). Then wrap up with the idea of probability (options that work for you over total options) and discuss how “and” and “or” change the probability.

Punnett Squares

Introduce the the topic of punnett squares. Use a simple example such as shown below:

A green pea plant (GG) is being crossed with a green pea plant (Gg).

	G	<u>G</u>	
G	GG	GG	<u>GenoType</u> = 2 GG: 2 <u>Gg</u> : 0 gg
<u>g</u>	Gg	Gg	Phenotype= 4 Green pea plants: 0 other color

Be sure to show students how the punnett square is just another way of determining permutations and combinations. Have students work in small groups to determine the outcomes of various crosses. Have them write their answers on the board in the classroom and discuss as a class. Below are various samples that could be used.

Genetics and Probability Pre/Post Test

1. If you roll two identical dice simultaneously, how many possible outcomes are there?
2. What if one of the dice is blue while the other is red. How many possible outcomes are there?
3. TT (tall) plant is crossed with a tt (short plant). What percentage of the offspring will be tall?
4. A Tt plant is crossed with a Tt plant. What percentage of the offspring will be short?
5. A heterozygous round seeded plant (Rr) is crossed with a homozygous round seeded plant (RR). What percentage of the offspring will be homozygous (RR)?

Blood Typing with Venn Diagrams (Discrete)

Lesson Objective: Students will understand how the 2 antigens of blood work together with the RH factor to form the different blood types.

Days of Instruction: 2

Day 1 - Pretest. Launch/Explore.

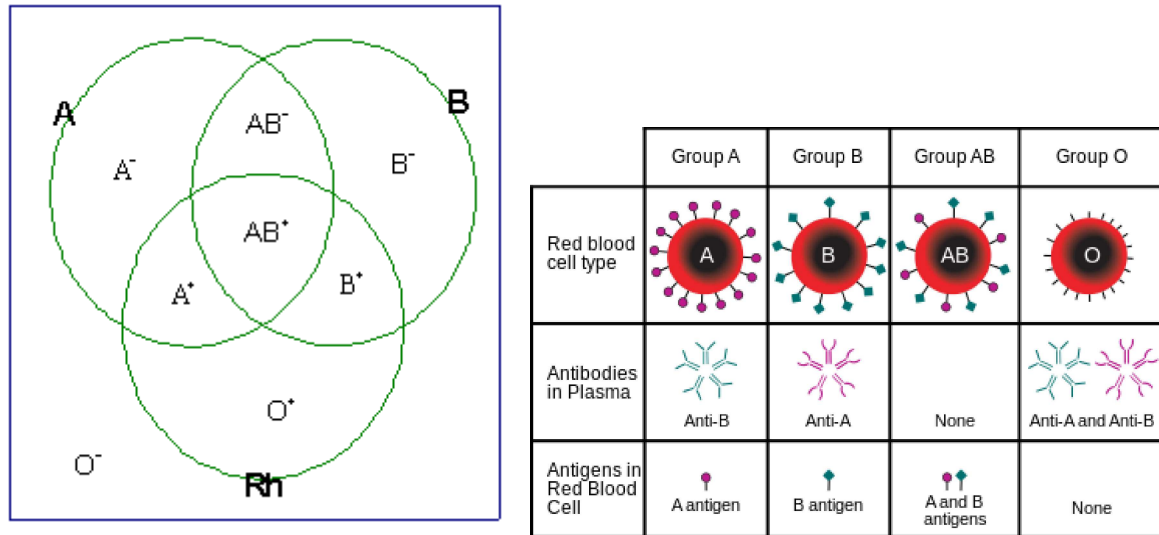
Day 2 - Compatible blood types. (unions, intersections, set notation) Posttest

Minnesota State Standard(s):

Life Science	Evolution in Living Systems	Variation within a species is the natural result of new inheritable characteristics occurring from new combinations of existing genes or from mutations of genes in reproductive cells.	9.4.3.2.1	Use concepts from Mendel's laws of segregation and independent assortment to explain how sorting and recombination (crossing over) of genes during sexual reproduction (meiosis) increases the occurrence of variation in a species.
Math	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.6	Describe the concepts of intersections, unions and complements using Venn diagrams. Understand the relationships between these concepts and the words AND, OR, NOT, as used in computerized searches and spreadsheets.

Venn Diagram with Blood Typing

Launch: Begin class by asking students if they know what their blood type is. Propose other questions that are relevant to blood typing such as: Why is it important to know your blood type if you have been injured and need blood? Who can receive which type of blood? How many different blood types are there?



Explore: Once the questions above have been discussed with the class, draw a Venn Diagram with 3 intersecting circles on the board. Label one circle “A”, one “B”, and the other “RH”. Allow the students an opportunity to see if they can fill out the Venn Diagram on their own. They will have intuitions about blood typing that will most likely be supported while they fill out the diagram.

Share: Have students share what they believe to be the correct way to fill out the Venn Diagram and discuss with the entire class.

Summarize: Use the information that students have discovered to go back and explain why blood typing is important in medical situations, what a “universal donor” is, what a “universal recipient” is, etc.

Blood Type Compatibility: Set Notation

Activity: Figure 2 shows the compatible blood types for B⁺. You should fill in the other seven diagrams showing compatible blood types for A⁺, A⁻, B⁻, AB⁺, AB⁻, O⁺, and O⁻. The instructor can choose to either have the students describe the shaded area in set notation or in regular words.

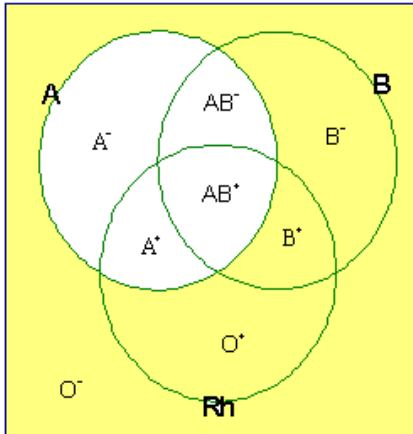
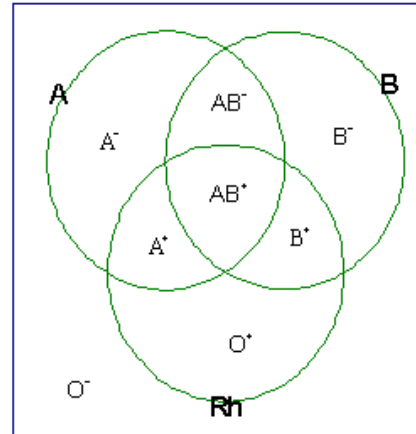
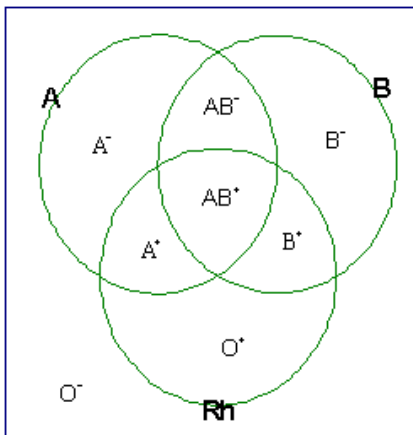


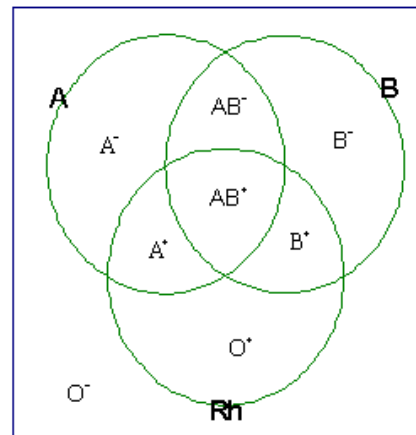
Fig. 2 (Blood Types compatible with B⁺)



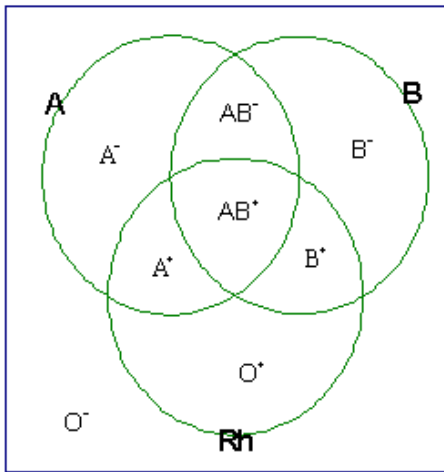
Blood Types compatible with B⁻



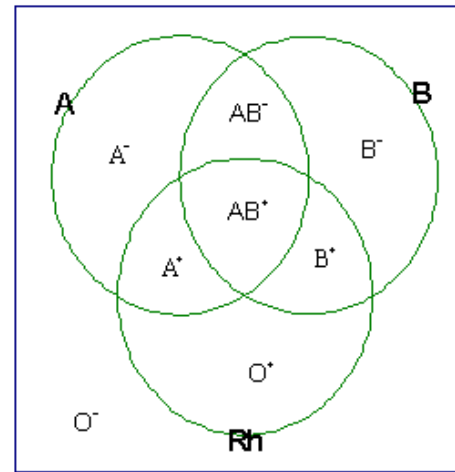
Blood Types compatible with O⁻



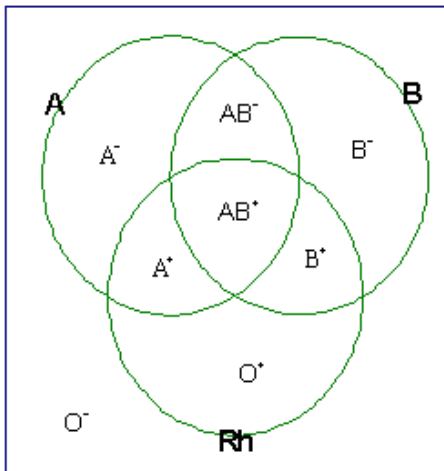
Blood Types compatible with O⁺



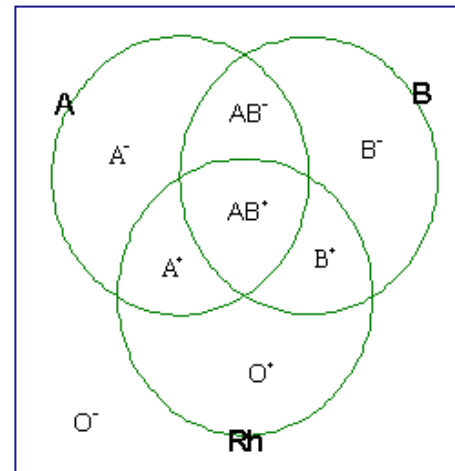
Blood Types compatible with A⁻



Blood Types compatible with A⁺



Blood Types compatible with AB⁻



Blood Types compatible with AB⁺

Password Security (Permutations)

Lesson Objective: To learn what permutations are and how to calculate permutations.

Days of Instruction: 3

Day 1 - Pretest. Launch/Explore.

Day 2 - Intro to permutations with ice cream cone sample. Group and class discussion.

Day 3 - How can we make stronger passwords?

Minnesota State Standard(s):

Math	Data Analysis & Probability	Calculate probabilities and apply probability concepts to solve real-world and mathematical problems.	9.4.3.1	Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.
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Launch: You walk out of your house and the door closes behind you then you ask yourself “Did I grab the key?”. You turn around and look through the sidelight by the door and see your keys on the table. It is always nice to have an extra key somewhere outside. When we bought house if had a keypad lockbox for an extra house key to be locked into. It consisted of 10 digits and you used three of the digits for the passcode of the lockbox. However being an old mechanical lockbox the order in which you enter the digits does not matter. So I wonder, how many three digit codes are possible?

EXPLORE: For many cell phones, the owner can set a 4-digit passcode to lock the device. Would it take someone very long to unlock your device? Working in groups (3 or 4 students per group), students will discuss and calculate various scenarios involving passwords, IDs, etc.

How many digits could you choose from for the first number of the pass code?

How many digits could you choose from for the second number of the pass code? Assume that the numbers can be repeated.

How many different 4-digit pass codes are possible? Explain how you got your answer.

How long (in hours) would it take someone to try every possible code if it takes three seconds to enter each possible code?

SHARE: Students will share their work while at the whiteboard with the class.

SUMMARIZE:

The Fundamental Counting Principle allows you to make these calculations. The process of choosing the four digits for the pass code or four digits for your school ID card involves a

sequence of events. You can multiply the number of choices that each element in a pass code or ID card can occur.

Permutations

Ice Cream Cones

Source: Maturra, R. (2013). MCTM 2013 spring conference ice cream cones & bowls [conference handout]. Department of Mathematics, St. Olaf College, Northfield, MN.

LAUNCH: Nearly everybody loves ice cream. What are the top three flavors for our class? We are going to discover how many different ice cream cones we can create with our top three flavors if it is important which scoop is on the top.

EXPLORE and SHARE:

In groups, students will be asked to consider task involving ice cream cones. Questions/tasks will be revealed one at a time. Sufficient time will be provided for each group to work and share thinking as a class.

ICE CREAM CONES

In an ice cream cone, it is important which scoop is on top. Thus, a vanilla-strawberry-chocolate cone is different from a strawberry-chocolate-vanilla cone. Also, you may not duplicate a flavor in a given cone. For example, vanilla-vanilla-chocolate is not a valid cone.

Suppose an ice cream shop has our three favorite ice cream flavors of ice cream _____, _____, and _____.

How many different three-scoop cones can you make using each of these flavors exactly once?

Suppose you want a four-scoop cone with your next favorite ice cream flavor _____. How many different cones can you make?

How many different cones can you make from 5 scoops of different flavors? 6 scoops? 7 scoops? 10 scoops?

How many different cones can you make from n scoops of different flavors? Explain how you know your answer is correct.

Suppose an ice cream shop serves 24 different flavors of ice cream.

How many different three-scoop cones can you make at the ice cream shop?

Note: As before, you cannot use the same flavor twice on one cone.

How many different four-scoop cones can you make at the ice cream shop?

SUMMARIZE:

Can we find a rule – first as a verbal description, then as a formula – for determining the number of different r -scoop cones you can make? (Base your rule on the 24-flavor ice cream shop.) How does our rule relate to our method for solving the previous cone problems?

Suppose you can make 156 different two-scoop cones at a certain ice cream shop. How many different flavors does this shop offer?

Suppose you can make 2,730 different three-scoop cones at a certain ice cream shop. How many different flavors does this shop offer?

How Can We Make Stronger Passwords?

Source: <http://robertkaplinsky.com/work/how-can-we-make-stronger-passwords/>

LAUNCH:

Are your passwords safe? One common type of password hacking (or cracking) is called “brute force” and involves using a computer to try every single possible combination of characters until it enters the correct one. For example, if a password was one character long and could only be an English lowercase letter, then there would only be 26 possible passwords. If a hacker tried each of them, he or she would be guaranteed to figure out the password by the 26th attempt. Where the math gets interesting is figuring out what password requirements make them more complex or more specifically, are more effective in increasing the total number of possible passwords a hacker has to try.

Potential password requirements include –

- May use lowercase characters
- May use uppercase characters
- May use digits
- May use symbols
- Minimum number of characters
- Maximum number of characters

EXPLORE:

The students start working in groups to agree on a certain set of password requirements and then calculate the total number of potential passwords based on those requirements. Students will be encouraged to play around with different password requirements to see how they affect the total number of potential passwords.

SHARE:

Each group shares their set of password requirements and calculated potential passwords. Students will discuss how the different password requirements affected the total number of passwords. Then as a whole class, students agree on a certain set of password requirements and then calculate the total number of potential passwords based on those requirements.

SUMMARIZE:

The Fundamental Counting Principle allows us to find the number of potential passwords and although we were able to find very large totals, a large total number of potential passwords does not imply that the passwords are secure. Given more time to explore, you would discover that the single most important factor in making a password more complex is the password’s length.

Password Security Pre/Post Test

1. Traci has decided to buy a pickup truck. Her choices include either a V-6 or a V-8 engine, a standard cab, an extended cab or a quad cab, a short box or a long box, and 2-wheel drive or 4-wheel drive. How many possible models does she have to choose from?
2. How many different ways can the letters of "BOOKKEEPER" be arranged?
3. How many ways can you line up 7 students in a lunch line?
4. The high school chorus has been practicing 8 songs, but there is time for only 4 of them at the spring concert. How many different orderings of 4 songs are possible?
5. How many basketball teams of 5 players can be formed from 12 players without regard to position played?
6. A license plate consists of a vowel (a, e, i, o, u) followed by two digits followed by 3 letters (letters may be uppercase or lowercase). How many different license plate are possible?